Philadelphia University
Department of Basic Sciences
Final Exam (Math 210106)

Student Name: ....
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Date: February 04, 2007
Instructor Name: ....
Section: ....

Remark: you have to read the following notes before you start answering the questions:

1 : The following are the Maclaurin series representations of some functions. You may need some of these to solve some of the questions through the exam.

MACLAURIN SERIES	INTERVAL OF CONVERGENCE
$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$	$-1 < x < 1$
$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$	$-\infty < x < \infty$
$\sin x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$	$-\infty < x < \infty$
$\cos x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}$	$-\infty < x < \infty$
$\ln(1+x) = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{x^k}{k}$	$-1 < x \leq 1$

2 : The following are the trigonometric functions of some important angles:

θ	$0°$	$30°$	$45°$	$60°$	$90°$
Radians	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞

3 : This is a multiple choice final exam of 50 marks. You have only 120 minutes to solve 26 paragraphs in 3 pages. Each paragraph has 2 marks. Write the correct answer for each paragraph in the table provided. Only the answers in the table will be graded.

1	2	3	4	5	6	7	8	9	10	11	12	13
14	15	16	17	18	19	20	21	22	23	24	25	26

♣ **GOOD LUCK** ♣

1. The rectangular coordinates of the points whose polar coordinates are $\left[6, \frac{\pi}{6}\right]$, are given by:

a. $\left(\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$ b. $(3\sqrt{3}, 3)$ c. $(0, 3)$ d. $(3, 3\sqrt{3})$

2. The equation $x^2 + y^2 - 6y = 0$ can be expressed in polar coordinates form as:

a. $r = -4\cos\theta$ b. $r = 6\cos\theta$ c. $r = 6\sin\theta$ d. $r = 4\sin\theta$

3. The directrix equation of the parabola $(y-3)^2 = 8(x-2)$ is given by:

a. $y = 0$ b. $y = 1$ c. $x = 1$ d. $x = 0$

4. The foci of the hyperbola $\frac{(x-1)^2}{16} - \frac{(y-1)^2}{9} = 1$ are:

a. $(-1, 3), (-1, -5)$ b. $(1, 3), (1, -1)$ c. $(2, -1), (-4, -1)$ d. $(6, 1), (-4, 1)$

5. The equation $x^2 - 5y^2 - 2x - 10y - 9 = 0$ represents an equation of:

a. ellipse b. parabola c. hyperbola d. circle

6. Let $f(x) = \frac{x-5}{x^2-1}$. Then $\int f(x)dx$ equals:

a. $3\ln|x+3| - 2\ln|x-3| + c$ b. $3\ln|x+1| - 2\ln|x-1| + c$
c. $3\ln|x+4| + 2\ln|x-4| + c$ d. $3\ln|x+2| + 2\ln|x-2| + c$

7. Evaluate the sum of $\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \cdots$

a. $\frac{1}{2}$ b. $\frac{3}{4}$ c. 1 d. None

8. One of the following is a *divergent* series:

a. $\sum_{k=1}^{\infty}(-1)^k\,\frac{1}{k}$ b. $\sum_{k=1}^{\infty}\frac{ke^{-k^2}}{4+e^{-k}}$ c. $\sum_{k=1}^{\infty}\frac{\sqrt[3]{k-1}}{k^3+1}$ d. $\sum_{k=1}^{\infty}(-1)^k\left(\frac{1}{2}\right)^k$

9. If a series $\sum_{k=0}^{\infty} a_k$ is an *absolutely convergent* series, then:

a. $\lim_{k\to\infty} a_k = 0$ b. $\sum_{k=0}^{\infty} a_k$ is a convergent series
c. $\sum_{k=0}^{\infty} a_k$ is a divergent series d. both (a) and (b) are true

10. The series $\sum_{k=1}^{\infty}(-1)^{k+1}\,\frac{4^k}{k^2}$ is:

a. absolutely convergent b. conditionally convergent
c. divergent d. None

11. The values of p such that $\sum_{k=1}^{\infty}\frac{p^k}{k}$ converges, are:

a. $-1 < p \leq 1$ b. $-1 < p < 1$ c. $-1 \leq p \leq 1$ d. $-1 \leq p < 1$

12. The interval of convergence of the series $\sum_{k=0}^{\infty}\frac{2^k}{k^2}(x+2)^k$ is:

a. $\left[\frac{3}{2}, \frac{5}{2}\right]$ b. $\left[-\frac{5}{2}, -\frac{3}{2}\right)$ c. $\left(\frac{3}{2}, \frac{5}{2}\right]$ d. $\left[-\frac{5}{2}, -\frac{3}{2}\right]$

13. The *Taylor* series representation of $g(x) = e^{-x}$ about $x = 1$ is given by:

a. $\sum_{k=0}^{\infty}(-1)(x+1)^k$ b. $\frac{1}{e}\sum_{k=0}^{\infty}\frac{(x+1)^k}{k!}$ c. $\sum_{k=0}^{\infty}(-1)(x+3)^k$ d. $\frac{1}{e}\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}(x-1)^k$

14. If $\sum_{k=0}^{\infty} a_k x^k$ has radius of convergence R such that $0 < R < \infty$, then the radius of convergence of $\sum_{k=0}^{\infty} a_k (x-c)^k$, for any constant c, will be:

a. R b. $R+c$ c. $R-c$ d. $\sqrt{R}$

15. The sum of the series $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}\pi^{2k+1}$ equals:

a. $-\pi$ b. $e-1$ c. $\frac{\pi}{2}$ d. $\sqrt{e}$

16. The *Maclaurin* series representation of the function $f(x) = \frac{2}{1+4x^2}$ has the form:

a. $\sum_{k=0}^{\infty} \frac{x^{2k+1}}{k!}$ b. $\sum_{k=0}^{\infty} \frac{(-1)^k x^{3k+1}}{k!}$ c. $\sum_{k=0}^{\infty} 2^{k+1} x^{2k}$ d. $\sum_{k=0}^{\infty} (-1)^k 2^{2k+1} x^{2k}$

17. Suppose that $\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6}$ and $\sum_{k=1}^{\infty} \frac{1}{k^4} = \frac{\pi^4}{90}$. Use these results to find $\sum_{k=1}^{\infty} \frac{3k^2-1}{k^4}$.

a. $\frac{\pi^2}{6} - \frac{\pi^4}{30}$ b. $\frac{\pi^2}{3} - \frac{\pi^4}{90}$ c. $\frac{\pi^2}{2} - \frac{\pi^4}{90}$ d. $\frac{\pi^2}{6} - \frac{\pi^4}{45}$

18. Find f_{xy} for the function $f(x,y,z) = x^3y^5z^7 + xy^2 + y^3z$.

a. $35x^3y^4z^6 + 3y^2$ b. $42x^3y^5z^5$ c. $21x^2y^5z^6$ d. $15x^2y^4z^7 + 2y$

19. Consider the function $f(x,y) = e^{2xy}$. Find the directional derivative of f at the point $(4,0)$ in the direction of the vector $\vec{\mathbf{u}} = \frac{-3}{5}\,\hat{\mathbf{i}} + \frac{4}{5}\,\hat{\mathbf{j}}$.

a. $\frac{32}{5}$ b. $-\frac{3}{\sqrt{10}}$ c. $-\frac{1}{\sqrt{2}}$ d. $-\frac{\sqrt{3}}{2}$

20. Let $f(x,y) = \frac{y}{x+y}$. The unit vector $\vec{\mathbf{u}}$ such that $D_{\mathbf{u}}f(2,3) = 0$ is:

a. $\frac{2}{\sqrt{13}}\,\hat{\mathbf{i}} + \frac{3}{\sqrt{13}}\,\hat{\mathbf{j}}$ b. $\frac{-2}{3}\,\hat{\mathbf{i}} - \frac{1}{3}\,\hat{\mathbf{j}}$ c. $\frac{3}{2}\,\hat{\mathbf{i}} - \frac{3}{2}\,\hat{\mathbf{j}}$ d. $\frac{2}{\sqrt{15}}\,\hat{\mathbf{i}} - \frac{3}{\sqrt{15}}\,\hat{\mathbf{j}}$

21. Find ∇w where $w = e^{-3y}\cos 4x$.

a. $-3e^{-3x}\sin 4y\,\hat{\mathbf{i}} + 4e^{-3x}\cos 4y\,\hat{\mathbf{j}}$ b. $-4e^{-3y}\sin 4x\,\hat{\mathbf{i}} - 3e^{-3y}\cos 4x\,\hat{\mathbf{j}}$
c. $-4e^{-4x}\cos 3y\,\hat{\mathbf{i}} - 3e^{-4x}\sin 3y\,\hat{\mathbf{j}}$ d. $3e^{-4y}\cos 3x\,\hat{\mathbf{i}} - 4e^{-4y}\sin 3x\,\hat{\mathbf{j}}$

22. If $\vec{\mathbf{v}} = 3\,\hat{\mathbf{i}} + 2\,\hat{\mathbf{j}} + \sqrt{3}\,\hat{\mathbf{k}}$, then $\|\vec{\mathbf{v}}\|$ equals:

a. 5 b. 7 c. 6 d. 4

23. The value of $\int\int_{\Omega} \cos(\pi xy)dydx$, over the region Ω bounded by $y=0$, $y=\frac{1}{2x}$, $x=1$ and $x=e$, equals:

a. e^2-1 b. π c. 12 d. $\frac{1}{\pi}$

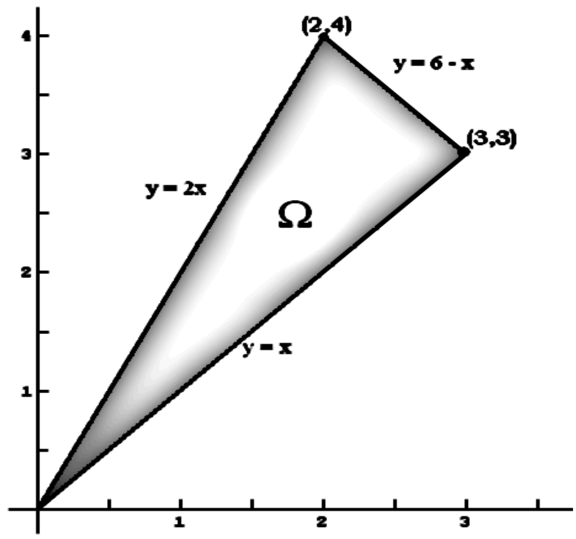
24. Evaluate $\int_0^1 \int_{\sqrt{x}}^1 \frac{3}{4+y^3}\,dydx$ by first changing the order of integration.

a. 5 b. e^4 c. $\ln\left(\frac{5}{4}\right)$ d. None

25. The value of $\int\int\int_G f(x,y,z)\,dx\,dy\,dz$ where $f(x,y,z) = 2x+y-z$ and $G = \{(x,y) : 0 \le x \le 2\,,\ -2 \le y \le 2\,,\ 0 \le z \le 2\}$ equals:

a. -7 b. -32 c. 108 d. 16

26. Let Ω be the region shown in the accompanying figure. Then $\int\int_{\Omega} f(x,y)\,dA$ can be expressed as:



a. $\displaystyle\int_0^3 \int_x^{2x} f(x,y)dydx.$

b. $\displaystyle\int_0^2 \int_x^{2x} f(x,y)dydx + \int_2^3 \int_x^{6-x} f(x,y)dydx.$

c. $\displaystyle\int_0^3 \int_{\frac{y}{2}}^{y} f(x,y)dxdy + \int_3^4 \int_{\frac{y}{2}}^{6-y} f(x,y)dxdy.$

d. both (b) and (c) are true.